conductivity were selected to be similar to those encountered in actual microcircuits. The radius of the energy source on the upper surface of the first cylinder was $1 \cdot 10^{-3}$ m and the specific power was $q_0 = 2 \cdot 10^5$ W/m^2.

The calculation determined the heat of the center of the source with respect to the lower surface of the last cylinder, for which the temperature field can be assumed uniform ($\lambda_5 = 20$ W/m · °K). A comparison of the values for the heating gives the following results: numerical method, 108.9°; proposed method including thermal flux nonuniformity at contact surfaces, 109°; proposed method without inclusion of nonuniformity, 112°; method of series-connected thermal resistances, 49°.

A comparison of the results shows that calculation by the proposed method can be made in some cases without consideration of thermal flux nonuniformity at contact surfaces, which leads to smaller errors than the assumption of isothermicity at those surfaces. In addition, an evaluation of the accuracy of the method was made on the basis of a comparison of the results of an experimental determination and of a calculation by the proposed method of the thermal resistance of actual microcircuits. The divergence of the results did not exceed the spread in the value of the thermal resistance of a microcircuit caused by instability in technical procedures.

NOTATION

P, power of energy source; q, flux density; λ_i, coefficient of thermal conductivity of the i-th parallelepiped; S_i, S_{i-1}, areas of the i-th and (i − 1)-th parallelepipeds; h_i, height of the i-th parallelepiped; $Bi_i = \alpha_i h_i / \lambda_i$, Biot number of the i-th parallelepiped; t_c, ambient temperature; $\vartheta = t - t_c$, heating.

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THEORY OF NEW KINETIC METHODS OF MEASURING THE MASS-TRANSFER PROPERTIES OF DISPERSED SOLIDS

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The solution of the generalized mass-transfer equations with variable coefficients is used as a basis for developing the theory of various new methods of determining the transfer characteristics of dispersed solids with finite moisture contents.

Methods of experimentally determining the mass-transfer characteristics of highly dispersed, moisture-containing solids employed at the present time are subject to a number of fundamental shortcomings which greatly reduce the accuracy and reliability of the results obtained. Practically all the methods of measuring mass-transfer characteristics are analogous to the corresponding heat-transfer methods. At the same time, mass transfer differs very considerably from heat transfer, despite the identical nature of the transfer equations. Thus, the use of any particular solution of the heat-conduction equation as a basis for the development of methods of determining mass-transfer coefficients is frequently unreliable, even though the corresponding thermophysical procedure has been thoroughly vindicated.

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In developing the theory of the proposed new methods the following basic characteristics of mass transfer were taken into account (in contradistinction to the majority of existing methods): first, the mass-transfer coefficients depend very considerably on mass content and temperature. Hence, the use of solutions derived from differential transfer equations with constant coefficients (which are completely acceptable in the case of thermal-conductivity measurements) is responsible for extremely serious errors, frequently exceeding 100%, when determining mass conductivity [1]. Procedures for measuring mass-transfer characteristics should therefore be based on the solutions of nonlinear transfer equations.

Secondly, the actual mechanism underlying the transfer of liquid and vapor in a dispersed body depends on the experimental conditions, and hence so do the transfer coefficients. This is precisely why the results of steady-state methods, despite their high degree of reliability, often deviate very considerably from the results obtained directly under kinetic conditions. This is a direct result of the general laws of mass transfer and does not represent a shortcoming of any particular method. Measurements of the mass-transfer coefficients should therefore always be carried out under conditions closely resembling the real conditions in which the resultant coefficients are to be quantitatively applied.

Thirdly, it is essential to make a sharp distinction between the mass-transfer coefficients (such as a_m and a_u) obtained on, respectively, using the chemical potential μ and the mass content u as mass-transfer potential. The μ(u) relationship determined by the absorption isotherm for any particular material is, in fact, usually very nonlinear ($c_m \neq$ const). Thus, for example, a_m and a_u, characterizing the rate of propagation of the μ and u surfaces of equal potential in the solid [2], are completely different, although they have identical dimensions. Similar differences apply to the other characteristics.

The proposed methods of determining the mass-transfer characteristics are based on an analysis of experimental data obtained during the convective drying of a dispersed solid in the form of an unbounded symmetrical plate of thickness $2l$. The differential drying equations for a case of this kind take the form [2]

$$\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(a_q \frac{\partial t}{\partial \tau}\right) + \frac{\varepsilon r c_m}{c_q} \cdot \frac{\partial \mu}{\partial \tau}, \tag{1}$$

$$\frac{\partial \mu}{\partial \tau} = \frac{\partial}{\partial x}\left(a_m \frac{\partial \mu}{\partial x} + a_m \delta \frac{\partial t}{\partial x}\right) \tag{2}$$

or

$$\frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(a_q \frac{\partial t}{\partial x}\right) + \frac{\varepsilon r}{c_q} \cdot \frac{\partial u}{\partial \tau}, \tag{1a}$$

$$\frac{\partial u}{\partial \tau} = \frac{\partial}{\partial x}\left(a_u \frac{\partial u}{\partial x} + a_u \delta_u \frac{\partial t}{\partial x}\right) \tag{2a}$$

subject to boundary conditions of the third kind

$$a_q c_q \rho \left(\frac{\partial t}{\partial x}\right)_l = \alpha_q (t_l - t_0), \tag{3}$$

$$a_m c_m \rho \left[\left(\frac{\partial \mu}{\partial x}\right)_l + \delta \left(\frac{\partial t}{\partial x}\right)_l\right] = \alpha_m (\mu_l - \mu_0) \tag{4}$$

or

$$a_u \rho \left[\left(\frac{\partial u}{\partial x}\right)_l + \delta_u \left(\frac{\partial t}{\partial \tau}\right)_l\right] = \alpha_u (u_l - u_0). \tag{4a}$$

The integration of these equations for variable transfer coefficients was carried out in [3]. The coefficients k_q and k_m (allowing for the distribution of potentials in the solid) enter into the solution of [3] in the form of cofactors attached to the corresponding characteristics of the material. As in the majority of investigations [4-6], we take a parabolic distribution of the potentials with respect to thickness, which corresponds to a value of k = 3. Possible deviations from the parabolic distribution have the effect that the transfer coefficients assume "effective" and not true values, although they only differ from the true values very slightly. Allowing for this, we derive the following from [3]:

$$\frac{\partial \bar{t}}{\partial \tau} = \frac{r c_m}{c_q} \cdot \frac{d\bar{\mu}}{d\tau} + \frac{3 a_q \alpha_q (t_0 - \bar{t})}{l (3 a_q c_q \rho + \alpha_q l)}, \tag{5}$$

$$\frac{d\bar{t}}{d\tau} = \frac{r}{c_q} \cdot \frac{d\bar{u}}{d\tau} + \frac{3a_q\alpha_q(t_0 - \bar{t})}{l(3a_qc_q\rho + \alpha_q l)}, \tag{5a}$$

$$\frac{d\bar{\mu}}{d\tau} = \frac{3a_m\alpha_m(3a_qc_q\rho + \alpha_q l)(\mu_0 - \bar{\mu}) + 3a_m\delta\alpha_q\alpha_m l(t_0 - \bar{t})}{l(3a_qc_q\rho + \alpha_q l)(3a_mc_m\rho + \alpha_m l)}, \tag{6}$$

$$\frac{d\bar{u}}{d\tau} = \frac{3a_u\alpha_u(3a_qc_q\rho + \alpha_q l)(u_0 - \bar{u}) + 3a_u\delta_u\alpha_u\alpha_q l(t_0 - \bar{t})}{l(3a_qc_q\rho + \alpha_q l)(3a_u\rho + \alpha_u l)} \tag{6a}$$

or using the Posnov criterion (number)

$$\frac{d\bar{\mu}}{d\tau} = \frac{3a_m\alpha_m(1 + \mathrm{Pn})(\mu_0 - \bar{\mu})}{l[3a_mc_m\rho(1 + \mathrm{Pn}) + \alpha_m l]}, \tag{7}$$

$$\frac{d\bar{u}}{d\tau} = \frac{3a_u\alpha_u(1 + \mathrm{Pn}_u)(u_0 - \bar{u})}{l[3a_u\rho(1 + \mathrm{Pn}_u) + \alpha_u l]}. \tag{7a}$$

In order to determine the characteristics of dispersed materials on the basis of Eqs. (5)-(7) several different modes of drying may be employed.

The first of these corresponds to $\mathrm{Bi}_q \to \infty$ and $\mathrm{Bi}_m \to \infty$, when transfer is determined by processes taking place inside the body, while $t_0 \to t_l$, $u_0 \to u_l$. In this case Eqs. (5)-(7) assume the form

$$\frac{d\bar{t}}{d\tau} = \frac{r}{c_q}\frac{d\bar{u}}{d\tau} + \frac{3}{l^2}a_q(t_0 - \bar{t}), \tag{8}$$

$$\frac{d\bar{\mu}}{d\tau} = \frac{3a_m}{l^2}[(\mu_0 - \bar{\mu}) + \delta(t_\theta - \bar{t})], \tag{9}$$

$$\frac{d\bar{u}}{d\tau} = \frac{3a_n}{l^2}[(u_0 - \bar{u}) + \delta_u(t_0 - \bar{t}), \tag{9a}$$

$$\frac{d\bar{\mu}}{d\tau} = \frac{3a_m}{l^2}(1 + \mathrm{Pn})(\mu_0 - \bar{\mu}), \tag{10}$$

$$\frac{d\bar{u}}{d\tau} = \frac{3a_u}{l^2}(1 + \mathrm{Pn}_u)(u_0 - \bar{u}). \tag{10a}$$

Comparison of (8)-(10) with (5)-(7) shows that from a theoretical point of view the first procedure is most convenient for measuring the heat- and mass-transfer characteristics inside the material, since the corresponding formulas easily yield the external heat- and mass-transfer characteristics, the experimental measurement of which presents serious difficulties. On carrying out a single experiment (drying a plate of dispersed material) by the first operating procedure, in fact, and then using Eq. (10) or (10a), we find

$$a_m(1 + \mathrm{Pn}) = \frac{l^2}{3[\mu_0 - \bar{\mu}]}\frac{d\bar{\mu}}{d\tau}, \tag{11}$$

$$a_u(1 + \mathrm{Pn}_u) = \frac{l^2}{3(u_0 - \bar{u})} \cdot \frac{d\bar{u}}{d\tau}. \tag{11a}$$

From the same experiment using Eq. (8) we may calculate the thermophysical characteristics:

$$a_q = \frac{l^2}{3(t_0 - \bar{t})}\left(\frac{d\bar{t}}{d\tau} - \frac{r}{c_q}\frac{d\bar{u}}{d\tau}\right), \tag{12}$$

$$\lambda_q = a_qc_q\rho = \frac{l^2\rho}{3(t_0 - \bar{t})}\left(c_q\frac{d\bar{t}}{d\tau} - r\frac{d\bar{u}}{d\tau}\right). \tag{12a}$$

Thus, from a single experiment we may determine all the characteristics over a wide range of mass contents, i.e., obtain the complete characteristics of the material.

The product $a_m(1 + \mathrm{Pn})$ itself is an important characteristic of the dispersed material, since it determines the mass transfer under the influence of differences in temperature and chemical potential. This characteristic might well be called adiabatic, in contrast to a_m, which characterizes isothermal mass transfer. Hence, the measurement of $a_m(1 + \mathrm{Pn})$ does not constitute a shortcoming of the method. At the same time, Eqs. (9) and (9a) enable us to determine a_m and δ separately. For this purpose we have to carry out two experiments with one sample under conditions differing in respect of t_0 or u_0, maintaining, of course, the basic requirements of the drying procedure. As a result of these two experiments we obtain the following from Eq. (9):

$$a_m = \frac{l^2\left[\frac{d\bar{\mu}_1}{d\tau}(t_{02}-t_2)-\frac{d\bar{\mu}_2}{d\tau}(t_{01}-\bar{t}_1)\right]}{3\left[(\mu_{01}-\bar{\mu}_1)(t_{02}-\bar{t}_2)-(\mu_{02}-\bar{\mu}_2)(t_{01}-\bar{t}_1)\right]}, \tag{13}$$

$$\delta = \frac{\frac{d\bar{\mu}_2}{d\tau}(\mu_{01}-\mu_1)-\frac{d\bar{\mu}_1}{d\tau}(\mu_{02}-\bar{\mu}_2)}{\frac{d\bar{\mu}_1}{d\tau}(t_{02}-\bar{t}_2)-\frac{d\bar{\mu}_2}{d\tau}(t_{01}-\bar{t}_1)}. \tag{14}$$

Analogous equations are obtained on using (9a) instead of (9).

Yet one more characteristic of the first procedure is of considerable interest. Equations (10)-(10a) were obtained without any special assumptions regarding a_m and Pn. If we consider these quantities as being constant, then by integrating (10) and (10a) we obtain

$$\bar{\mu}-\mu_0 = A_1\exp\left\{-\frac{3a_m}{l^2}(1+\mathrm{Pn})\tau\right\}, \tag{15}$$

$$\bar{u}-u_0 = A_2\exp\left\{-\frac{3a_u}{l^2}(+1+\mathrm{Pn}_u)\tau\right\}\cdot \tag{15a}$$

In other words, for constant transfer coefficients the difference in the chemical potentials of the liquid in a dispersed solid and the surrounding medium (or the corresponding u) depends exponentially on τ.

Equations (15)-(15a) are entirely analogous to the laws of cooling which express the behavior of a solid body in a medium at constant temperature when a regular thermal situation is established [7]. It was shown in [8, 9] by solving the heat- and mass-transfer equations with constant coefficients that both mass-transfer and heat-transfer processes were capable of passing into a "regular" stage, in which the temperature and mass content varied exponentially with time. The results of the present investigation show, however, that t and u only assume an exponential relationship for constant internal heat- and mass-transfer coefficients, and such cases are not typical of highly dispersed porous solids.

The experimental measurement of the time-dependent quantities $\bar{t}$, $\bar{u}$, $d\bar{t}/d\tau$, $d\bar{u}/d\tau$ in (5)-(14) during the drying of dispersed solids is quite easy and may be carried out very accurately. Much greater (though surmountable) difficulties arise in keeping high values of Bi_m and Bi_q during the drying process. It is important to satisfy this condition experimentally because, inter alia, many existing methods of determining the mass-transfer characteristics of moist porous solids [6, 10] also assume that the experiments are carried out for large values of Bi_m and Bi_q, although the authors in question did not specifically stipulate this. The application of the corresponding computing formulas may produce seriously distorted results if these conditions are unsatisfied. We may note, in passing, that the computing formula of the method described in [6] may be expressed as a particular case of our own equations, subject to the constancy of the transfer coefficients, a parabolic distribution of moisture content, and the condition $\mathrm{Pn} \ll 1$.

We see from the foregoing that, despite the simplicity of the computing formulas, the experimental realization of the first procedure is fraught with difficulties. In particular, the creation of high Bi_m and Bi_q compels us to work with samples of great thickness, which extends the period of the experiment and averages its results over the whole thickness of the sample. In addition to the first procedure it is therefore desirable to use other forms of drying from which these experimental difficulties are excluded.

Let us consider a second procedure in which Bi_m and Bi_q have values of the order of unity. Substituting for the mass-content $u = m/m_0$ and executing some obvious transformations, (7) and (7a) may be written

$$\frac{3l\rho c_m}{\alpha_m}\frac{dm}{d\tau}+\frac{l^2}{a_m(1+\mathrm{Pn})}\frac{dm}{d\tau} = 3m_0c_m(\mu_0-\bar{\mu}), \tag{16}$$

$$\frac{3l\rho}{\alpha_u}\frac{dm}{d\tau}+\frac{l^2}{a_u(1+\mathrm{Pn}_u)}\frac{dm}{d\tau} = 3m_0(u_0-\bar{u}). \tag{16a}$$

In order to determine the mass-transfer characteristics of the material by (16) and (16a) we must carry out two experiments under the same conditions but for different thicknesses of the material. In the first experiment (with small l values) it is easy to calculate α_m or α_u from (16)-(16a) for a particular sample and particular conditions, since the middle terms of the equations are negligibly small. The second experiment is carried out with a fairly thick sample. Substituting α_m and α_u derived from the first experiment into (16) and (16a) it is easy to calculate $\alpha_\mathrm{m}(1+\mathrm{P_n})$.

The necessity of carrying out two experiments with each sample is, of course, a disadvantage of the second method of determining the mass-transfer characteristics, since it increases both the time taken and the error committed. However, on carrying out the experiments by the second procedure there is no need to create very rigorous drying conditions, and this greatly simplifies the experiment. The error committed in carrying out the two experiments may be greatly reduced by increasing the thickness of the sample in the second experiment, so that the term containing α_m is small by comparison with the second term in (16) and α_m may be measured quite roughly in the first experiment.

Also of considerable interest is a third procedure in which the dispersed solid sample is "thin" with respect to heat transfer ($Bi_q \to 0$) but not with respect to mass transfer. For the experimental realization of this arrangement it is sufficient to take a sample of the same thickness as in the second procedure, but of a small cross section, with thin moisture-insulation of the side walls and bottom. The mass (moisture) flow then remains one-dimensional. Heat transfer through the side walls and bottom of the sample equalizes the temperature, which is equivalent to an increase in the thermal conductivity of the sample and a reduction in Bi_q. For calculating the mass-transfer characteristics in the third measuring procedure, all the laws associated with the second procedure may be used, but with due allowance for the fact that the temperature difference in the sample is small, and correspondingly $Pn \to 0$. Thus, after carrying out experiments and calculations analogous to those characterizing the second procedure, but under the conditions of the third, we may find the isothermal and not the adiabatic characteristics of internal mass transfer.

In order to determine the effective thermophysical characteristics of dispersed solids (allowing for mass transfer), it is convenient to pass to a fourth procedure in which $Bi_q \to \infty$. In this procedure, in fact, the heat-transfer equation takes the form (5), i.e., it enables us to calculate a_q or λ_q very easily. The experimental realization of the fourth procedure is much simpler than the first.

The higher the value of Bi_q, in fact, the closer is the surface temperature to the temperature of the ambient. Thus, if instead of t_0 we substitute the sample surface temperature into (5), we find that (5) remains valid even when the condition $Bi_q \gg 1$ is not satisfied, i.e., for comparatively slow modes of drying. A change in surface temperature during the experiment will not distort the results, since in deriving the general formulas (5)-(7) constancy of t_0 was nowhere assumed.

In deriving all the foregoing equations we considered that the material under test experienced hardly any shrinkage ($l \approx \text{const}$). However, if it becomes impermissible to neglect shrinkage during the drying process, the computing formulas remain valid, but different values of l will have to be substituted into them at different stages of the experiment, and this greatly complicates the experiments and calculations. However, this shortcoming, like any possible changes in the pore structure of the material under different shrinkage conditions, applies equally to all known complex methods of investigation in which a wide range of moisture contents of the material is embraced by a single experiment.

The foregoing cases do not, of course, exhaust all possible methods of measuring the mass-transfer and thermophysical characteristics. An analysis of the general equations (5)-(7) will probably enable us to distinguish other cases of drying which are easy to execute experimentally and serve as a basis for calculating the characteristics of the materials under test. In conclusion, we note that all the proposed methods are rapid and comprehensive, since from the results of a single experiment we may determine the characteristics of the material over a wide range of mass contents.

NOTATION

t, temperature; μ, chemical potential; u, mass content ($\bar{t}$, $\bar{\mu}$, $\bar{u}$, average values over the volume of the body; t_0, μ_0, u_0, values in the ambient); τ, time; x, coordinate; a_q, thermal diffusivity; λ_q, thermal conductivity; a_m, a_u, mass potential conductivity coefficients; δ, δ_u, thermal-gradient coefficients; ε, phase-transformation criterion; c_q, c_m, specific heat and mass capacities; ρ, density; r, specific heat of vaporization; α_q, α_m, external heat- and mass-transfer coefficients; A_1, A_2, integration constants; Pn, Posnov number; Bi_q, Bi_m, heat- and mass-transfer Biot numbers.

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A CALCULATION METHOD FOR TANGENTIAL SPRAY DRYERS

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A method is presented for calculating the design parameters of a cylindrical spray dryer with tangential heat-carrier input; the relationships are analyzed.

Spiral flows of interacting phases are used in drying equipment because they set up a centrifugal pattern, which allows one to raise the concentration of the dispersed component, increase the relative speed of the phases, and thus accelerate the heat and mass transfer.

Here we examine a mathematical model for droplet interaction with a spiral gas flow in order to define the particle dynamics in relation to chamber design, with the object of providing basic concepts for design purposes.

The motion of a droplet in such a chamber is a complex curvilinear one in a spatial velocity distribution; the motion is affected by numerous different forces and other effects [1, 2]. The following external forces act on a particle in the general case: gravitation, reactive evaporation, molecular attraction, electrostatic, electromagnetic, and gasdynamic.

The latter includes the aerodynamic resistance, the counterpressure, the force due to turbulent mass transfer, and the Magnus force, the last arising by rotation of a particle around its axis.

It is impossible to reflect the combined action of all these forces fully in the differential equation, so various assumptions must be made.

We consider the motion of the heat carrier along a spiral line whose pitch is uniform along the length and radius, with the motion taken as stationary at an average velocity and as not involving rapid turbulent exchange. Radial leakage and secondary eddies are neglected, as is the loss of small particles toward the center of the chamber. It is assumed that the centrifugal force is purely radial, while the tangential and axial velocities of the particle and carrier are equal at each instant. In addition, we incorporate the change in droplet mass as well as the effects of the radial centrifugal force and viscous resistance.

The centrifugal force is defined by

$$F_{\text{c.f.}} = m_{\text{p}} \frac{w_{\text{p}}^2}{r} . \tag{1}$$

This force produces radial motion of the particle (separation speed), and the result is a resistance force exerted by the gas:

$$F_{\text{r.f.}} = \psi s \frac{(v - v_{\text{p}})^2 \gamma_1}{2q} , \tag{2}$$

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